A Spectator-Quark-Model for the Photoproduction of Kaons

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Abstract

A simple model for the photoproduction of kaons off protons with a lambda hyperon in the final state is presented. In a quark model, the interaction is modelled by the pair-creation of the (anti-) strange quarks in the final state which recombine with the three quarks of the proton to form the lambda and kaon. The calculated scattering cross sections for photon energies up to $E_\gamma = 1.9$ GeV are compared to experiment. The pair-creation process is found to have a significant contribution to the total cross section.

I. INTRODUCTION

In the last years, there has been renewed interest in the photoproduction of strange particles off protons. This is especially due to powerful new facilities (SAPHIR at ELSA (Bonn) [1] , CLAS at CEBAF in the near future) which provide new data with better statistics. The process $\gamma + p \to K^+ + \Lambda/\Sigma^0$ is a useful tool to study strangeness and its production in hadronic matter. From the measured cross sections one can extract strong coupling constants and magnetic moments of the produced hyperons. Of special interest are polarization observables to study the spin-dependences of the reaction. Many efforts have been made to describe the process $\gamma + p \to K^+ + \Lambda/\Sigma^0$ for medium energies (up to photon energies $E_\gamma = 2$ GeV). Good results for the cross sections have been achieved with various isobaric models, of which the work of Adelseck et al. [2] is the latest example. However, Adelseck et al. have been unable to explain the polarization data, like the recoil polarization of the hyperon. In addition, their calculated total cross section rises for photon energies larger than 1.4 GeV, in contrast to experiment [1,3]. One might expect that the observed recoil polarization is a direct consequence of the quark structure of hadronic matter, as has been proposed by Miettinen [4] for the strangeness production in pp collisions. In the naive quark model, where a baryon is composed of three quarks, the spin of the Λ is carried by the strange quark alone, in contrast to the Σ^0, where u and d quark couple to a spin triplet. Thus, it is a challenging task to explain the recoil polarization in the framework of a simple quark model.

This paper describes a simple model of the process $\gamma + p \to K^+ + \Lambda$ for energies up to $E_\gamma = 1.9$ GeV. The baryons are composed of three quarks ($\Lambda = (uds)$, $p = (uud)$) and the K^+ meson of a quark-antiquark pair ($u\overline{s}$). The only contribution being considered (see fig.1) describes the reaction by a pair-creation of the strange quark and antiquark which recombine with the up quark to the K^+ and the up and down quarks to the Λ, respectively. The pair-creation process, however, does not describe the Σ^0 production. This can be seen by considering forward scattering, where obviously only the $M_{S_{12}} = 0$ component of the $S_{12} = 1$ (ud) state contributes. Thus, the Σ^0 production amplitude is suppressed by a factor 1/3. The baryon states are described by an integral over quark states and a Gaussian function and the meson state by that over quark-antiquark states and a Gaussian function [5,6].

II. THE MODEL

The antisymmetrized baryon wave function has the following structure, e.g. for the proton

$$|N_s(P_N)\rangle = N_\alpha \sqrt{2P_N^0} \int \frac{d^3p_\rho}{(2\pi)^3} \int \frac{d^3p_\lambda}{(2\pi)^3} \frac{1}{\sqrt{2p_1^0\, 2p_2^0\, 2p_3^0}} \mathcal{R}_\alpha(p_\rho, p_\lambda)\, \chi_N^F \chi_N^C$$

$$\cdot \left[\left[|\vec{p}_1, s_1\rangle \otimes |\vec{p}_2, s_2\rangle\right]^{s_{12}} \otimes |\vec{p}_3, s_3\rangle\right]^{s} \qquad (1)$$

with

$$\begin{aligned}
\vec{p}_\rho &= \frac{m_2}{m_1+m_2}\,\vec{p}_1 - \frac{m_1}{m_1+m_2}\,\vec{p}_2 \\
\vec{p}_\lambda &= \frac{1}{m_1+m_2+m_3}\,\left(m_3\,(\vec{p}_1+\vec{p}_2) - (m_1+m_2)\vec{p}_3\right) \quad .
\end{aligned} \qquad (2)$$

The various functions and parameters will be explained below. After symmetrizing the wave function with respect to the interchange of any two quarks in the baryon, it suffices to calculate only graph (a) in fig.1 and multiply by 3, which gives diagram (d.) (see appendix D). The graph invokes the idea that the quark pair may form a diquark. Indeed, after correctly symmetrizing, one can integrate in the $\mathcal{T}$ matrix, see below, over $\vec{p}_\rho$, thus replacing $|\vec{p}_1\rangle\ |\vec{p}_2\rangle$ by $|\vec{p}_{12}\rangle$. In our model, the internal dynamics of the (ud) pair does not affect the scattering process. From now on, the index 1 denotes the spectator (ud) pair. We thus can write for the hadron wave functions (note that for Λ production the diquark has spin 0):
The proton wave function is

$$|N_{s_2}(P_N)\rangle = N_\alpha\sqrt{2P_N^0}\int \frac{d^3p}{(2\pi)^3}\,\frac{1}{\sqrt{2p_1^0\,2p_2^0}}\,\mathcal{R}_\alpha(p)\ \chi_N^F\chi_N^C\ |\vec{p}_1\rangle\,|\vec{p}_2, s_2\rangle \qquad (3)$$

with the (di-)quark states

$$\begin{aligned}
|\vec{p}_1\rangle\ &= \left|\frac{m_1}{m_1+m_n}\vec{P}_N + \vec{p}\right\rangle = \tilde{a}^\dagger(p_1)\ |0\rangle \\
|\vec{p}_2, s_2\rangle\ &= \left|\frac{m_n}{m_1+m_n}\vec{P}_N - \vec{p}, s_2\right\rangle = a_{s_2}^\dagger(p_2)\ |0\rangle \quad ,
\end{aligned} \qquad (4)$$

the Gaussian function

$$\mathcal{R}_\alpha(p) = e^{-\alpha^2p^2} \quad , \quad p = |\vec{p}\,|$$

and χ^F, χ^C the flavour and colour functions with the correct symmetry for the ground state. Analogously, we have for the Λ wave function

$$\langle Y_{s_s}(P_Y)| = N_\beta\,\sqrt{2P_Y^0}\int \frac{d^3p'}{(2\pi)^3}\,\frac{1}{\sqrt{2p'^0_1\,2p_s^0}}\,\mathcal{R}_\beta(p')\ \chi_Y^F\chi_Y^C\ \langle\vec{p}_1\,'|\ \langle\vec{p}_s, s_s| \qquad (5)$$

with

$$\begin{aligned}
\langle\vec{p}_1\,'|\ &= \left\langle\frac{m_1}{m_1+m_s}\vec{P}_Y + \vec{p}\,'\right| = \langle 0|\ \tilde{a}(p_1') \\
\langle\vec{p}_s, s_s|\ &= \left\langle\frac{m_s}{m_1+m_s}\vec{P}_Y - \vec{p}\,', s_s\right| = \langle 0|\ a_{s_s}(p_s)
\end{aligned} \qquad (6)$$

and the K meson state is written as

$$\langle K(P_K)| = \sum_{s_{\overline{s}} s_2'} C^{00}_{\frac{1}{2}s_{\overline{s}}\frac{1}{2}s_2'}\, N_\gamma \sqrt{2P_K^0} \int \frac{d^3p''}{(2\pi)^3} \frac{1}{\sqrt{2p^0_{\overline{s}}\, 2p'^0_2}} \mathcal{R}_\gamma(p'')\, \chi^F_K \chi^C_K \;\langle \vec{p_{\overline{s}}}, s_{\overline{s}}|\; \langle \vec{p_2}\,', s_2'| \tag{7}$$

with

$$\begin{aligned} \langle \vec{p_{\overline{s}}}, s_{\overline{s}}| &= \left\langle \frac{m_s}{m_s+m_n}\vec{P}_K + \vec{p}\,'', s_{\overline{s}} \right| = \langle 0|\; b_{s_{\overline{s}}}(p_{\overline{s}}) \\ \langle \vec{p_2}\,', s_2'| &= \left\langle \frac{m_n}{m_s+m_n}\vec{P}_K - \vec{p}\,'', s_2' \right| = \langle 0|\; a_{s_2'}(p_2') \quad . \end{aligned} \tag{8}$$

The decomposition of the field operators $\Psi, \overline{\Psi}$ is given in appendix B. The normalization reads

$$< \vec{k}, s|\vec{k}\,', s' > = \;(2\pi)^3\,(2k^0)\,\delta^3(\vec{k}-\vec{k}\,')\,\delta_{ss'} \quad . \tag{9}$$

This leads to the standard normalization of the hadronic states (see appendix A).

There are at least five parameters: the three oscillator parameters of the proton, the Λ and the K^+, respectively, and the constituent quark masses of the strange (s) and non-strange (n) quarks. One may introduce an effective mass of the spectator diquark being less than twice the mass of the n-quark, which is favoured by many diquark models [7]. In fact, the (ud) pair forming a scalar diquark (better: diquark correlation) is postulated by many authors, see [8] for a review on that subject. However, we will see that the variation of those parameters in a reasonable range does not greatly affect the prediction of the model.

The following $\mathcal{T}$ matrix is calculated. For Λ production the spin of the spectator diquark is $s_1 = s_1' = 0$, and thus the z-components of the proton and Λ spin are $s = s_2$ and $s' = s_s$, respectively.

$$\mathcal{T} = \left\langle Y_{s_s}(P_Y), K(P_K) \;\middle|\overline{\Psi}(0)\, ie\,\hat{Q}\,\gamma^\mu\,\Psi(0)\,A_\mu(0)\middle|\; N_{s_2}(P_N), (\vec{k},\lambda) \right\rangle \quad . \tag{10}$$

With the commutator relations of the creation and annihilation operators (see appendix B) we find for real photons ($\lambda = 1,\, 2$) (without colour and flavour factors):

$$\begin{aligned} \mathcal{T} = {} & N_\alpha N_\beta N_\gamma \sqrt{2P^0_N}\sqrt{2P^0_Y}\sqrt{2P^0_K} \\ & \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2p^0_s\, 2p^0_{\overline{s}}}} \mathcal{R}_\alpha(p)\mathcal{R}_\beta(p'(p))\mathcal{R}_\gamma(p''(p)) \\ & \sum_{s_{\overline{s}}} C^{00}_{\frac{1}{2}s_{\overline{s}}\frac{1}{2}s_2} \left(\overline{u}_{s_s}(p_s)\, ie\hat{Q}\,\gamma^k\, v_{s_{\overline{s}}}(p_{\overline{s}})\,\epsilon^{(\lambda)}_k\right) \quad . \end{aligned} \tag{11}$$

For the operator one gets

$$\mathcal{O} := \sum_{s_{\overline{s}}} C^{00}_{\frac{1}{2}s_{\overline{s}}\frac{1}{2}s_2}\ \overline{u}_{s_s}(p_s)\,\gamma^k\,v_{s_{\overline{s}}}(p_{\overline{s}})\,\epsilon_k^{(\lambda)} \tag{12}$$

$$= \sum_{s_{\overline{s}}} C^{00}_{\frac{1}{2}s_{\overline{s}}\frac{1}{2}s_2}\ (-1)\sqrt{p_s^0+m_s}\,\sqrt{p_{\overline{s}}^0+m_s}\ \chi^\dagger_{s_s}\left(\sigma^k+\frac{\vec{\sigma}\vec{p}_s}{p_s^0+m_s}\,\sigma^k\,\frac{\vec{\sigma}\vec{p}_{\overline{s}}}{p_{\overline{s}}^0+m_s}\right)\tilde{\chi}_{s_{\overline{s}}}\,\epsilon_k^{(\lambda)} \quad .$$

In the center-of-mass system, this can be written as

$$\mathcal{O} = \sum_{s_{\overline{s}}} C^{00}_{\frac{1}{2}s_{\overline{s}}\frac{1}{2}s_2}\ \sum_{i=1}^{19} 4\pi\,\tilde{c}_i\,p^{t_i}\,P_K^{v_i}\,k^{u_i}$$

$$\chi^\dagger_{s_s}\left[Y_{\tilde{l}_{2_i}}(\hat{P}_K)\otimes\left[\left[Y_{l_{2_i}}(\hat{p})\otimes\left[\sigma^{a_i}\otimes Y_{l_{1_i}}(\hat{p})\right]^{k_i}\right]^{a_i}\otimes Y_{\tilde{l_{1_i}}}(\hat{k})\right]^{\tilde{k}_i}\right]^1_k\ \tilde{\chi}_{s_{\overline{s}}}\,\epsilon_k^{(\lambda)} \tag{13}$$

with momentum-dependent coefficients $\tilde{c}_i = \tilde{c}_i(p_s^0, p_{\overline{s}}^0)$.
Now define

$$\begin{aligned}
\tilde{m}_1 &= \frac{m_1}{m_1+m_n} \quad , \quad \tilde{m}_n = \frac{m_n}{m_1+m_n}\\
\tilde{\tilde{m}}_1 &= \frac{m_1}{m_1+m_s} \quad , \quad \tilde{\tilde{m}}_n = \frac{m_n}{m_s+m_n}\\
\delta &= \sqrt{\alpha^2+\beta^2+\gamma^2}\\
\alpha_1 &= \frac{1}{\delta}\left(-\beta^2\tilde{m}_1+\gamma^2\tilde{m}_n\right) \quad , \quad \alpha_2 = \frac{1}{\delta}\left(\beta^2\tilde{\tilde{m}}_1+\gamma^2\tilde{\tilde{m}}_n\right) \quad .
\end{aligned} \tag{14}$$

A decomposition with Clebsch-Gordan coefficients, integration over $d\Omega_p$ (where we neglect the angular dependence of p_s^0 and $p_{\overline{s}}^0$), identification of the z-direction with $\hat{P}_K$ and partial summation gives, now for the general case of a spectator-diquark spin $S_1 = 0$ or 1

$$\begin{aligned}
\mathcal{T} &= \left\langle Y_{s'}(P_Y), K(P_K)\ \middle|\overline{\Psi}(0)\,ie\,\hat{Q}\,\gamma^\mu\,\Psi(0)\,A_\mu(0)\middle|\ N_s(P_N), (\vec{k},\lambda)\right\rangle\\
&= \sqrt{2P_N^0}\sqrt{2P_Y^0}\sqrt{2P_K^0}\,N_\alpha N_\beta N_\gamma\\
&\quad\cdot\exp\left(-k^2(\beta^2\tilde{m}_1^2+\gamma^2\tilde{m}_n^2-\alpha_1^2)\right)\\
&\quad\cdot\exp\left(-P_K^2(\beta^2\tilde{\tilde{m}}_1^2+\gamma^2\tilde{\tilde{m}}_n^2-\alpha_2^2)\right)\\
&\quad\cdot\exp\left(-k\,P_K\,\cos\,\theta_K\,(-2\beta^2\tilde{m}_1\tilde{\tilde{m}}_1+2\gamma^2\tilde{m}_n\tilde{\tilde{m}}_n-2\alpha_1\alpha_2)\right)\\
&\quad\cdot\sum_{i=1}^{19}\mathcal{I}_i\,c_i\,k^{u_i}\,P_K^{v_i}\\
&\quad\cdot\sum_{s_1s_2s_sm_s\tilde{m}_k\tilde{m}_{1_i}\tilde{m}_{2_i}}\sqrt{\frac{2\tilde{l}_{2_i}+1}{4\pi}}\,\delta_{\tilde{m}_{2_i}0}\\
&\qquad\qquad\sqrt{\frac{2\tilde{l}_{1_i}+1}{4\pi}}\sqrt{\frac{(\tilde{l}_{1_i}-\tilde{m}_{1_i})!}{(\tilde{l}_{1_i}+\tilde{m}_{1_i})!}}\,P_{\tilde{l}_{1_i}}^{\tilde{m}_{1_i}}(\cos\,\theta_K)\\
&\quad\cdot\mathcal{O}^{i;s_1s_2s_sk}_{\tilde{m}_{1_i}\tilde{m}_{2_i}m_s\tilde{m}_k}\\
&\quad\cdot\frac{ie}{3}\,N_C\,N_{SF}
\end{aligned} \tag{15}$$

with

$$
\begin{aligned}
\mathcal{O}^{i;s_1 s_2 s_s k}_{\tilde{m}_{1_i}\tilde{m}_{2_i} m_s \tilde{m}_k} = 4\pi \ \frac{\hat{k}_i}{\hat{a}_i}\,(-1)^{l_{1_i}} & \\
& \cdot \hat{a}_i\,(-1)^{1-s_s+\frac{1}{2}} \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & a_i \\ s_s & -s_2 & -m_s \end{pmatrix} \\
& \cdot \hat{\tilde{k}}_i\,(-1)^{a_i-\tilde{l}_{1_i}+\tilde{m}_k} \begin{pmatrix} a_i & \tilde{l}_{1_i} & \tilde{k}_i \\ m_s & \tilde{m}_{1_i} & -\tilde{m}_k \end{pmatrix} \\
& \cdot \sqrt{3}\,(-1)^{\tilde{l}_{2_i}-\tilde{k}_i+k} \begin{pmatrix} \tilde{l}_{2_i} & \tilde{k}_i & 1 \\ \tilde{m}_{2_i} & \tilde{m}_k & -k \end{pmatrix} \\
& \cdot \sqrt{2}\,(-1)^{S_1-\frac{1}{2}+s} \begin{pmatrix} S_1 & \frac{1}{2} & \frac{1}{2} \\ s_1 & s_2 & -s \end{pmatrix} \\
& \cdot \sqrt{2}\,(-1)^{S_1-\frac{1}{2}+s'} \begin{pmatrix} S_1 & \frac{1}{2} & \frac{1}{2} \\ s_1 & s_s & -s' \end{pmatrix}
\end{aligned}
\tag{16}
$$

and the integral over the momentum p is (with the angular dependence of $p_s^0, p_{\bar{s}}^0$ neglected)

$$
\begin{aligned}
\mathcal{I}_i = \frac{1}{(2\pi\delta)^3} \ \int dp\, p^{2+t_i} \ \exp(-\alpha^2\, p^2) & \left(-\frac{\sqrt{p_s^0+m_s}\sqrt{p_{\bar{s}}^0+m_s}}{\sqrt{2p_s^0\, 2p_{\bar{s}}^0}} \right) \\
& \cdot \left\{ \frac{1}{(p_s^0+m_s)(p_{\bar{s}}^0+m_s)} \right\}_{i>1} \quad .
\end{aligned}
\tag{17}
$$

The spin-flavour and colour coefficients are (see appendix D)

$$
N_{SF} = \sqrt{\frac{3}{2}} \quad , \tag{18}
$$

$$
N_C = \frac{1}{\sqrt{3}} \quad , \tag{19}
$$

and the coefficients c_i are constant factors.
The averaged differential scattering cross section in the c.m.-frame is as usual

$$
\begin{aligned}
\frac{d\sigma}{d\Omega} &= \frac{1}{64\,\pi^2\, E_{CM}^2}\ \frac{p_f}{p_i}\ \overline{|\mathcal{T}|^2} \\
\text{with } p_i = k = P_N &= \frac{1}{2}(E_{CM} - \frac{M_N^2}{E_{CM}}) \\
p_f = P_K = P_Y &= \frac{1}{2E_{CM}}\,((E_{CM}^2 - M_K^2 - M_Y^2)^2 - 4M_K^2 M_Y^2)^{\frac{1}{2}} \quad .
\end{aligned}
\tag{20}
$$

III. RESULTS

For the following values of the parameters we find a semi-quantitative description of the differential and total scattering cross sections:

$$\begin{aligned} m_n = 300 \text{ MeV} \quad &, \quad m_s = 500 \text{ MeV} \quad , \quad m_1(\Lambda) = 500 \text{ MeV} \quad , \\ \alpha = 0.6 \text{ fm} \quad &, \quad \beta = 0.4 \text{ fm} \quad , \quad \gamma = 0.3 \text{ fm} \quad . \end{aligned} \tag{21}$$

The values of the constituent quark masses are in a range adopted in most quark models (see e.g. [9]). The spectator diquark mass is chosen a little less than the sum of u and d quarks [7]. The oscillator parameters decribing the extension of the hadrons are in a physical acceptable region (see also [9] for models predicting the nucleon and pion quark-core radii). The radii of the strange hadrons may follow from the relation motivated by an oscillator potential: $\frac{r_2}{r_1} = \sqrt{\frac{M_1}{M_2}}$.
Fig.2 to 4 show the results of our model in comparison to experimental data taken from [1]. The broad maximum of the total cross section between $E_\gamma = 1.0$ GeV and 1.4 GeV as well as the probable decrease for energies greater than 1.4 GeV [3] is qualitatively reproduced very well. However, the calculated curve is too low by a factor of about 2.5. In fact, a variation of the parameters cannot increase the calculated results. Of course a correct quantitative decription of the experimental data was not expected. This is due to the simplicity of the model; we simply neglect the coupling of the photon to the spectator quarks. Although the calculated differential cross sections are too small, the backward scattering ($\cos(\theta_K) < 1$) is described almost quantitatively. The rise of the differential cross section with increasing $\cos(\theta_K)$ is reproduced, too.
Fig.5 shows the calculated total cross section for different values of the spectator diquark mass m_1. Fig.6 shows the model prediction for different proton oscillator parameters. Apparently, the variation of the results is modest, once the parameters have been chosen in a physical acceptable range. It is interesting to see that a smaller diquark mass has almost the same effect as a smaller nucleon oscillator parameter, as is expected for a larger binding energy.

IV. SUMMARY AND OUTLOOK

We developed a simple quark model to describe qualitatively and semi-quantitatively the differential and total cross sections of the reaction $\gamma + p \rightarrow K^+ + \Lambda$ for photon energies up to 1.9 GeV. The only contribution considered is the coupling of the photon to the strange quark and antiquark in the final state, which recombine with the spectator quarks of the proton. Apparently, this process has a large contribution to the cross section. It would be interesting to study the contribution of the spectator quarks to the reaction, which cannot be calculated in our model. A main drawback of the model is that it is not able to calculate the recoil polarization, which is zero since the following relation holds for the $\mathcal{T}$ matrix:

$$\mathcal{T}_{ss'\lambda} = (-1)^{s+s'+\lambda}\, \mathcal{T}^{\,*}_{-s-s'-\lambda} \quad . \tag{22}$$

To obtain a non-vanishing polarization one may include hadronic resonances.

In a future calculation, the photoproduction of both Λ and Σ^0 will be examined in a fully relativistic model. The transition matrix will be calculated in the Mandelstam formalism [10], and the hadrons are described as relativistically bound states of quarks through Bethe-Salpeter amplitudes. In this framework, all three diagrams, where the photon couples to all possible internal quark lines, are taken into account. This gives rise to interferences. In addition, the amplitudes are properly boosted, which cannot be neglected in kaon-photoproduction.

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FIGURES

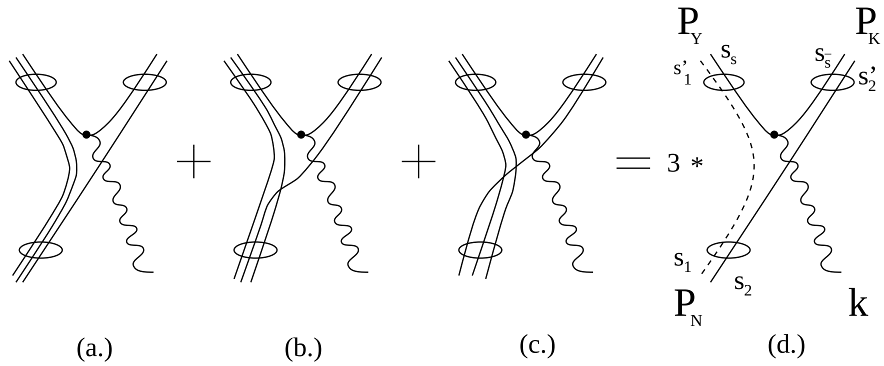

FIG. 1. Photoproduction of mesons by quark-antiquark pair-creation; the photon couples to the (anti-)strange quarks, the u-d-quark pair and the u-quark are spectator particles; (d.) is the only diagram after correctly symmetrizing the baryonic wave functions.

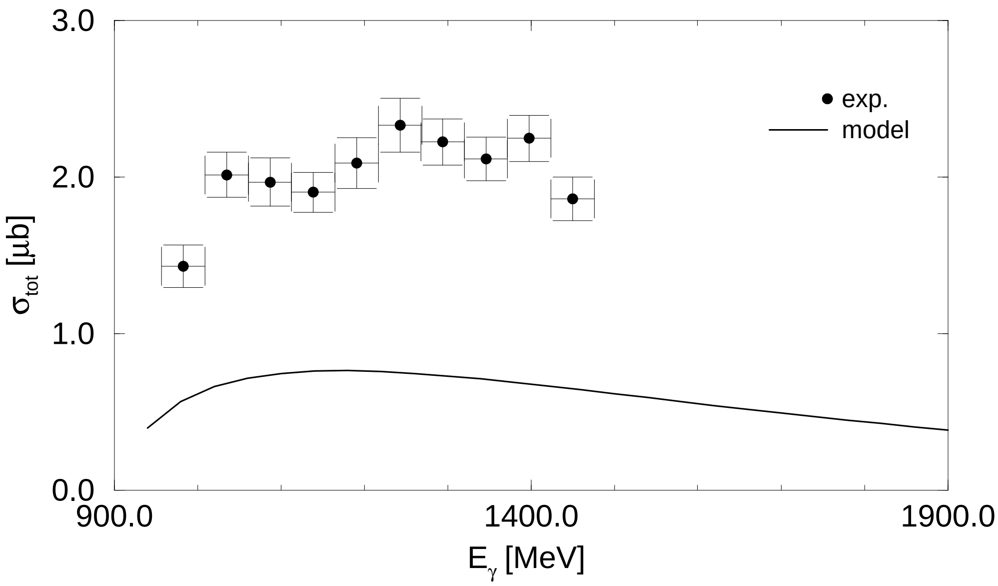

FIG. 2. The total cross section of the Λ production; experimental data are taken from [1]

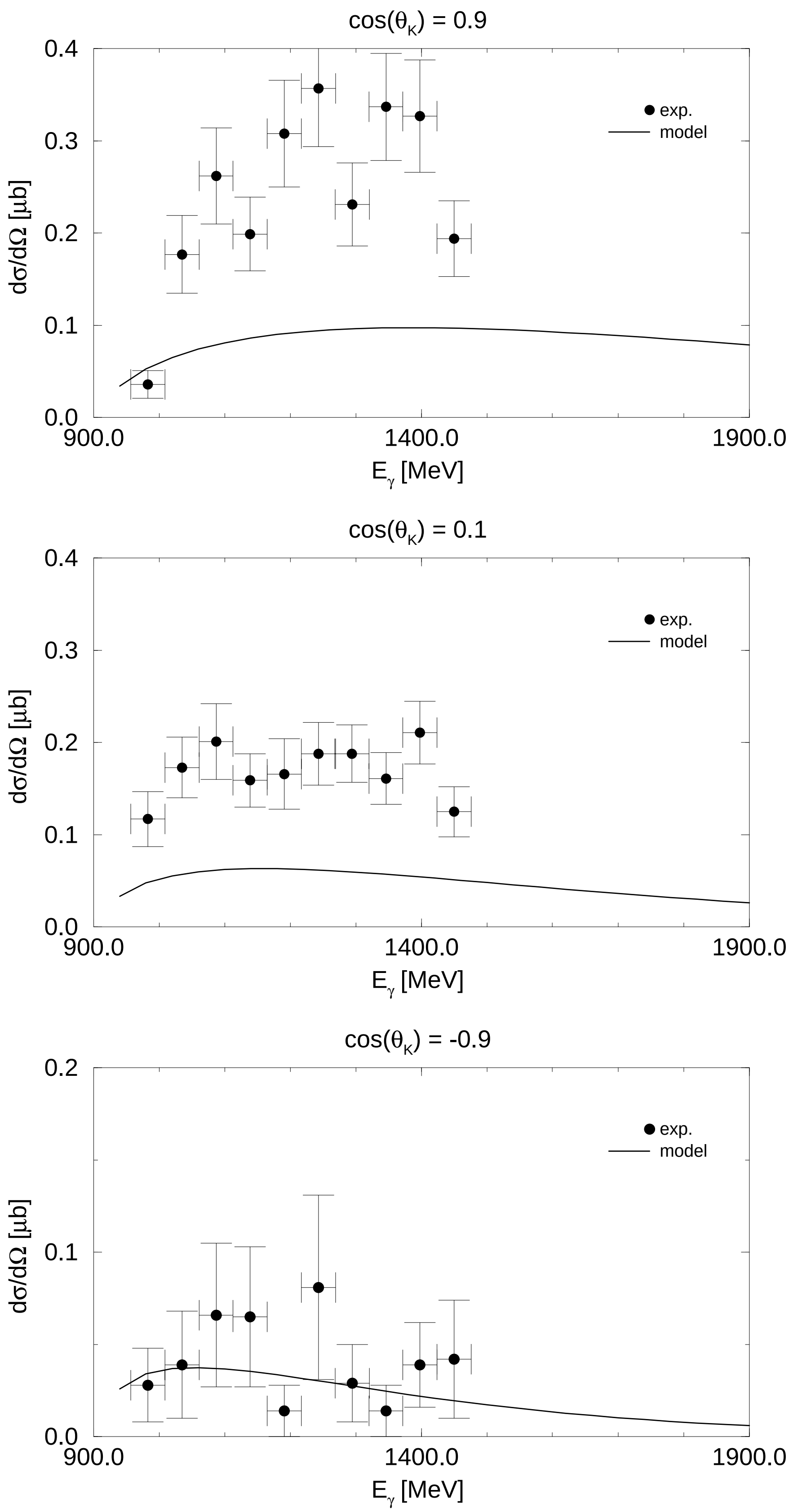

FIG. 3. The differential cross section of the Λ production for three different scattering angles; experimental data are taken from [1]

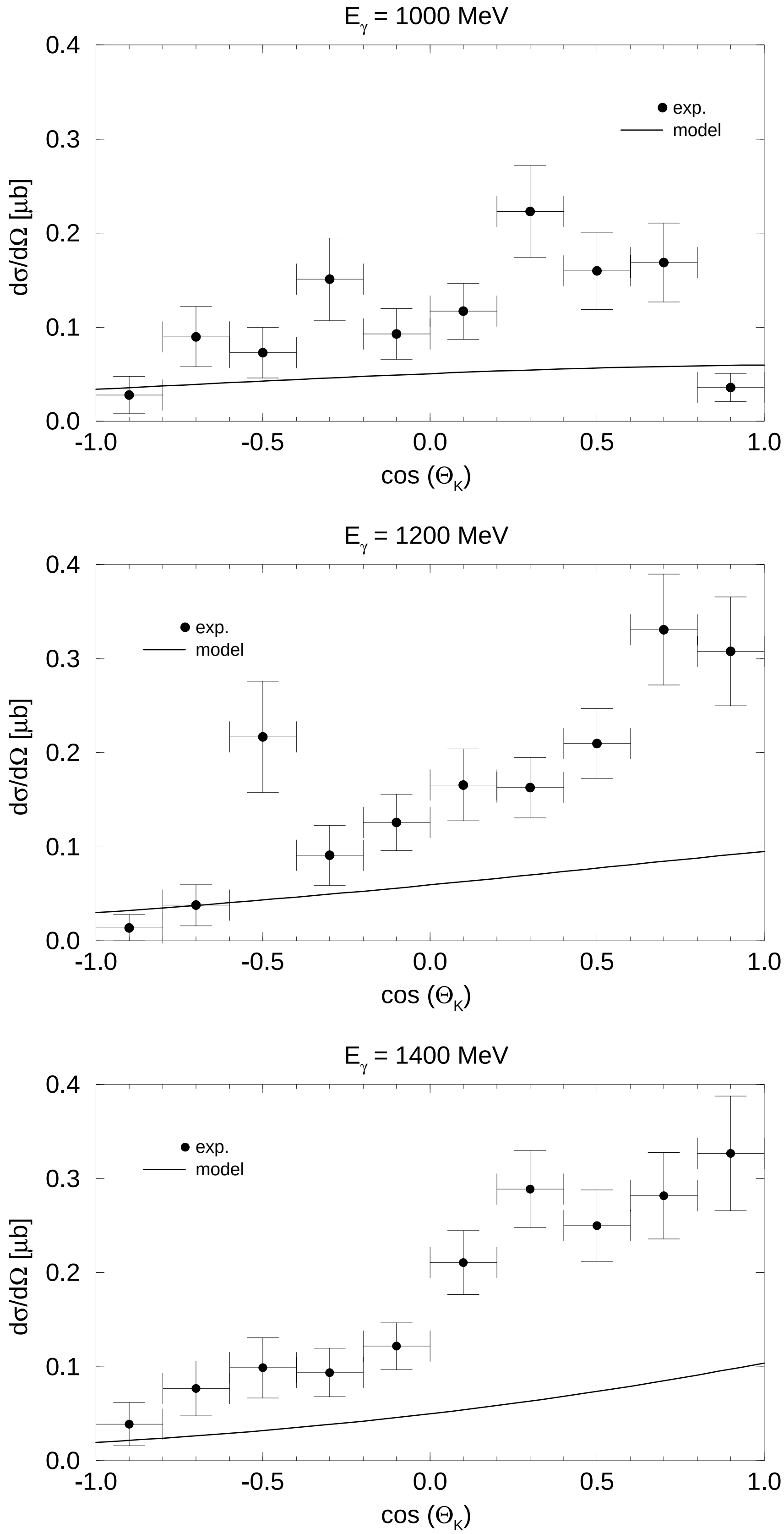

FIG. 4. The differential cross section of the Λ production for three different photon energies; experimental data are taken from [1]

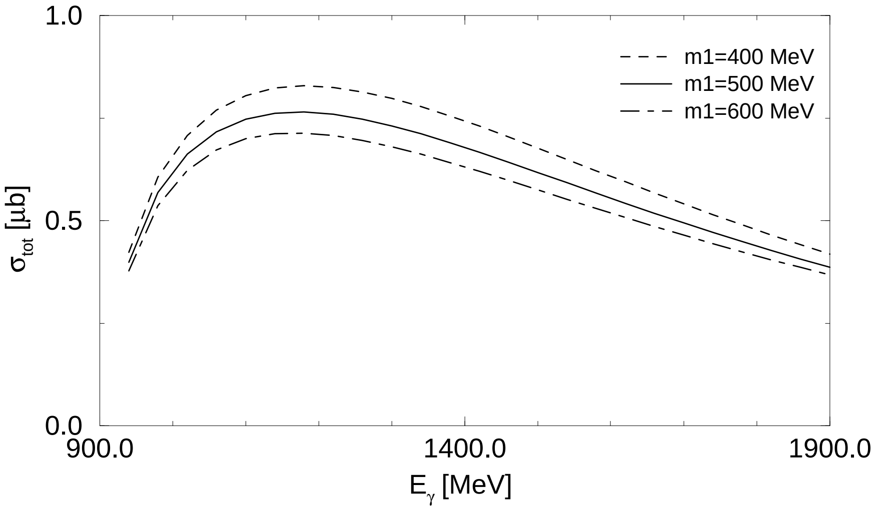

FIG. 5. The total cross section of the Λ production for three different values of the spectator diquark mass m_1

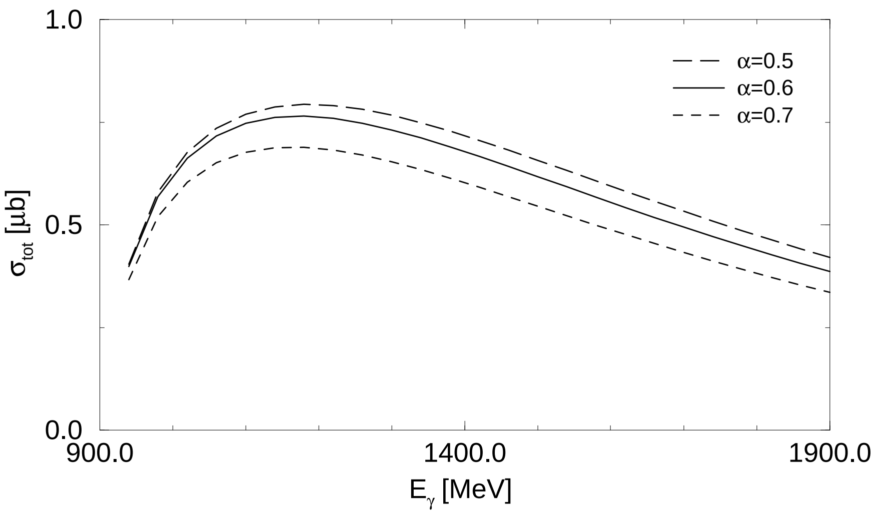

FIG. 6. The total cross section of the Λ production for three different values of the nucleon oscillator parameter α

APPENDIX A: NORMALIZATION OF THE BOUND STATES

The states

$$|N_s(P_N)\rangle = N_\alpha \sqrt{2P_N^0}\ \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2p_1^0\,2p_2^0}} \mathcal{R}_\alpha(p)\chi_N^F \chi_N^C\ |\vec{p}_1\rangle\,|\vec{p}_2, s\rangle \tag{A1}$$

are normalized according to:

$$\langle N_s(P_N)\,|N_{s'}(P'_N)\rangle = (2\pi)^3\,(2P_N^0)\delta^3(\vec{P}_N - \vec{P}'_N)\,\delta_{ss'}\ \delta_{FF'} \quad . \tag{A2}$$

It follows:

$$N_\alpha = 4\pi\,\alpha^{\frac{3}{2}}\ \left(\frac{2}{\pi}\right)^{\frac{1}{4}} \quad . \tag{A3}$$

Analogous for N_β and N_γ.

APPENDIX B: DECOMPOSITION OF FIELD OPERATORS, COMMUTATOR RELATIONS

The quark and photon field operators are

$$\begin{aligned}
\Psi(0) &= \int \frac{d^3\tilde{\tilde{p}}}{(2\pi)^3} \frac{1}{2\tilde{\tilde{p}}^0} \sum_{\tilde{\tilde{s}}} \Big(v_{\tilde{\tilde{s}}}(\tilde{\tilde{p}})\, b^\dagger_{\tilde{\tilde{s}}}(\tilde{\tilde{p}}) + u_{\tilde{\tilde{s}}}(\tilde{\tilde{p}})\, \tilde{a}_{\tilde{\tilde{s}}}(\tilde{\tilde{p}}) \Big) \\
\overline{\Psi}(0) &= \int \frac{d^3\tilde{p}}{(2\pi)^3} \frac{1}{2\tilde{p}^0} \sum_{\tilde{s}} \Big(\overline{v}_{\tilde{s}}(\tilde{p})\, b_{\tilde{s}}(\tilde{p}) + \overline{u}_{\tilde{s}}(\tilde{p})\, \tilde{a}^\dagger_{\tilde{s}}(\tilde{p}) \Big) \\
\{b_{\tilde{s}}(\tilde{p}), b^\dagger_{\tilde{\tilde{s}}}(\tilde{\tilde{p}})\} &= \{a_{\tilde{s}}(\tilde{p}), a^\dagger_{\tilde{\tilde{s}}}(\tilde{\tilde{p}})\} = (2\pi)^3\,(2\tilde{p}^0)\,\delta^3(\tilde{\vec{p}} - \tilde{\tilde{\vec{p}}})\,\delta_{\tilde{s}\tilde{\tilde{s}}} \\
A_\mu(0) &= \int \frac{d^3k}{(2\pi)^3} \frac{1}{2k^0} \sum_{\lambda=1}^{2} \epsilon_\mu^{(\lambda)}(k)\,(a^{(\lambda)}(k) + a^{(\lambda)\,\dagger}(k)) \\
\rightarrow\ A_\mu(0)\,\Big|(\vec{k},\lambda)\Big\rangle &= \epsilon_\mu^{(\lambda)}(k)\ |0\rangle \\
[a^{(\lambda)}(k), a^{(\lambda')\,\dagger}(k')] &= (2\pi)^3\,(2k^0)\,\delta^3(\vec{k} - \vec{k}\,')\,\delta_{\lambda\lambda'} \quad , \quad \lambda = 1\,, 2
\end{aligned} \tag{B1}$$

and the bosonic operators fulfill:

$$[\tilde{a}_0(p), \tilde{a}_0^\dagger(p')] = (2\pi)^3\,(2p^0)\,\delta^3(\vec{p} - \vec{p}\,') \quad . \tag{B2}$$

APPENDIX C: COORDINATES

The (di-)quark coordinates are

$$
\begin{aligned}
\vec{p}_1 &= \tilde{m}_1\, \vec{P}_N + \vec{p} \\
\vec{p}_2 &= \tilde{m}_n\, \vec{P}_N - \vec{p} \\
\vec{p}_1\,' &= \tilde{\tilde{m}}_1\, \vec{P}_Y + \vec{p}\,' \\
\vec{p}_s &= \frac{m_s}{m_1 + m_s}\, \vec{P}_Y - \vec{p}\,' \\
\vec{p}_{\bar{s}} &= \frac{m_s}{m_s + m_n}\, \vec{P}_K + \vec{p}\,'' \\
\vec{p}_2\,' &= \tilde{\tilde{m}}_n\, \vec{P}_K - \vec{p}\,'' \quad .
\end{aligned}
\tag{C1}
$$

From $\vec{p}_1 = \vec{p}_1\,'$ and $\vec{p}_2 = \vec{p}_2\,'$ follows

$$
\begin{aligned}
\vec{p}\,' &= \tilde{m}_1 \vec{P}_N - \tilde{\tilde{m}}_1 \vec{P}_Y + \vec{p} \\
\vec{p}\,'' &= \tilde{\tilde{m}}_n \vec{P}_K - \tilde{m}_n \vec{P}_N + \vec{p} \quad .
\end{aligned}
\tag{C2}
$$

APPENDIX D: SYMMETRY COEFFICIENTS

The three-quark wave functions of the baryons are

$$
\Psi(1,2,3) = \psi^S_{space}(1,2,3)\; (\chi_{spin} \otimes \phi_{flavour})^S(1,2,3)\; \chi^A_{colour}(1,2,3) \quad . \tag{D1}
$$

The spin-flavour function is symmetric under the interchange of any two quarks:

$$
\begin{aligned}
|p\rangle \;&=\; \frac{1}{\sqrt{2}}\; \left(\chi_{M_S}\; [[n\; n]^1 n]^{\frac{1}{2}} + \chi_{M_A}\; [[n\; n]^0 n]^{\frac{1}{2}} \right) \\
|\Lambda\rangle \;&=\; \frac{1}{2}\, \chi_{M_S}\; \left([[sn]^{\frac{1}{2}}\; n]^0 \;+\; [[ns]^{\frac{1}{2}}\; n]^0 \right) \\
&\quad + \chi_{M_A}\; \left(-\frac{1}{\sqrt{12}}\; [[sn]^{\frac{1}{2}}\; n]^0 \;+\; \frac{1}{\sqrt{3}}\; [[nn]^0\; s]^0 \;+\; \frac{1}{\sqrt{12}}\; [[ns]^{\frac{1}{2}}\; n]^0 \right) \\
|\Sigma\rangle \;&=\; -\frac{1}{\sqrt{12}} \chi_{M_S} [[ns]^{\frac{1}{2}} n]^1 + \frac{1}{\sqrt{3}} \chi_{M_S} [[nn]^1 s]^1 - \frac{1}{\sqrt{12}} \chi_{M_S} [[sn]^{\frac{1}{2}} n]^1 \\
&\quad + \frac{1}{2} \chi_{M_A} [[ns]^{\frac{1}{2}} n]^1 - \frac{1}{2} \chi_{M_A} [[sn]^{\frac{1}{2}} n]^1
\end{aligned}
\tag{D2}
$$

with e.g.

$$
\begin{aligned}
[[nn]^0 n]^{\frac{1}{2}} &= \frac{1}{\sqrt{2}} (ud - du) u \\
[[nn]^1 n]^{\frac{1}{2}} &= \frac{1}{\sqrt{6}} ((ud + du) u - 2uud) \\
[[nn]^1 s]^1 &= \frac{1}{\sqrt{2}} (ud + du) s \quad .
\end{aligned}
\tag{D3}
$$

The χ_{M_A}, χ_{M_S} are the (in quarks 1-2) mixed (anti-)symmetric spin functions:

$$\begin{aligned} \chi_{M_A} &= \left[\left[\frac{1}{2} \otimes \frac{1}{2}\right]^0 \otimes \frac{1}{2}\right]^S \\ \chi_{M_S} &= \left[\left[\frac{1}{2} \otimes \frac{1}{2}\right]^1 \otimes \frac{1}{2}\right]^S \quad . \end{aligned} \tag{D4}$$

For the Λ production one gets from

$$3 \left\langle \frac{1}{\sqrt{3}} \chi_{M_A} \; [[nn]^0 s]^0 \,\middle|\, \frac{1}{\sqrt{2}} \chi_{M_A} \; [[nn]^0 n]^{\frac{1}{2}} \right\rangle \tag{D5}$$

the factor $N_{SF} = \sqrt{\frac{3}{2}}$, for the Σ production from

$$3 \left\langle \frac{1}{\sqrt{3}} \chi_{M_S} \; [[nn]^1 s]^1 \,\middle|\, \frac{1}{\sqrt{2}} \chi_{M_S} \; [[nn]^1 n]^{\frac{1}{2}} \right\rangle \tag{D6}$$

the same factor $N_{SF} = \sqrt{\frac{3}{2}}$. The colour factor results from the colour singlet functions of the hadrons

$$\begin{aligned} |\mathrm{Baryon}\rangle_{col} &= \frac{1}{\sqrt{6}}(rgb - rbg + gbr - grb + brg - bgr) \\ |\mathrm{Meson}\rangle_{col} &= \frac{1}{\sqrt{3}}(\overline{r}r + \overline{g}g + \overline{b}b) \end{aligned} \tag{D7}$$

to

$$N_C = 3 \, \frac{1}{\sqrt{3}} \, \frac{1}{\sqrt{6}} \, 2 \, \frac{1}{\sqrt{6}} = \frac{1}{\sqrt{3}} \quad . \tag{D8}$$

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